Electrostatics of two dielectric balls and a point charge

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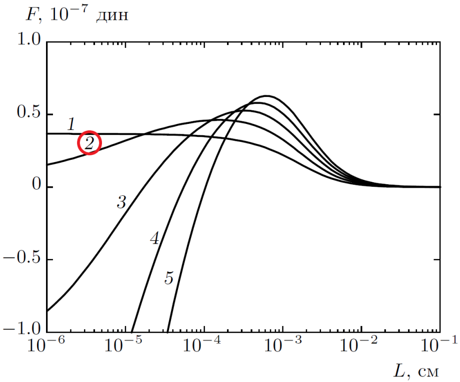
NPP-2021 Workshop

Applications

- Dusty plasmas and electrolytes
- Molecular clusters
- Water droplets in clouds
- Extraterrestrial atmospheres
- Toner particles

Interaction force

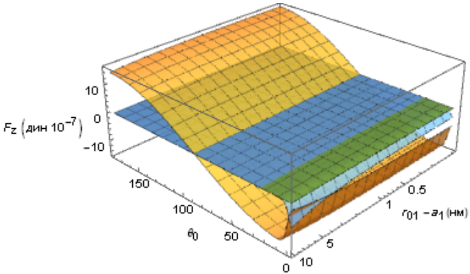


Force versus separation distance for different charge ratios while $a_1 = a_2 = 10\,\mu\text{m}$, $q_1 = 10^3 e$, $\varepsilon_{1,2} = 25$ *[Munirov, Filippov, JETP, 2013]*

Curve 2 is for $q_1/q_2 = 1.5$.

In this case no attraction takes place between the balls as $L \to 0$.

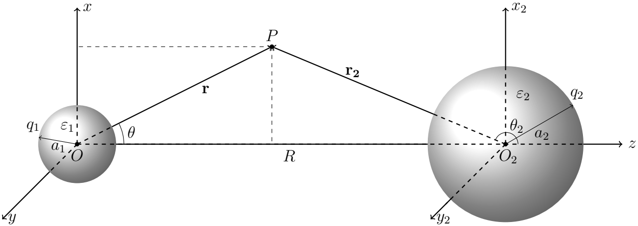
Interaction force



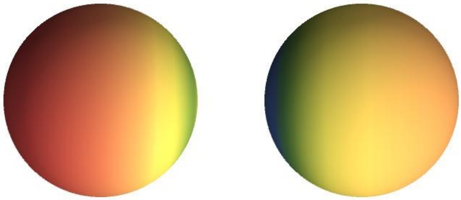
Force along z-axis on the balls versus the point charge location for $q_0 = 0.05q_1$, $L = 10^{-3}a_2$

- $F_{1z} < 0$ if $\theta_0 \lesssim 90^\circ$ (yellow surface)
- $F_{2z} < 0$ if $\theta_0 \lesssim 25^\circ$ (blue surface)
- If $\theta_0 \sim 0$, attraction is greater between q_0 and the nearest ball

Surface charge distribution for different geometries

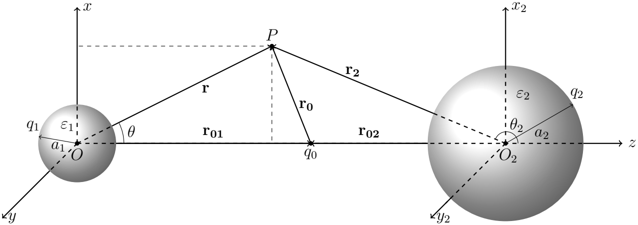


Geometrical representation for two spheres

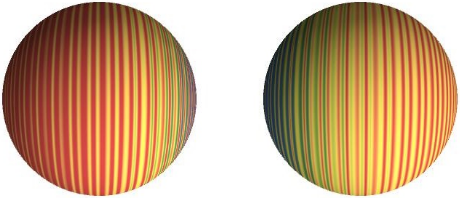


Surface charge distribution

Surface charge distribution for different geometries



Geometrical representation for two spheres and a point



Surface charge distribution (divergence is evident)

Charge density

Contribution of a point charge to the surface charge density:

$$\sigma_{1,b}^{(0)} = -\frac{q_0}{4\pi a_1 r_{01}} \sum_{n=0}^{\infty} \frac{n\left(\varepsilon_1 - \varepsilon\right)}{n\varepsilon_1 + \left(n+1\right)\varepsilon} \left(2n+1\right) \left(\frac{a_1}{r_{01}}\right)^n P_n\left(\mu\right).$$

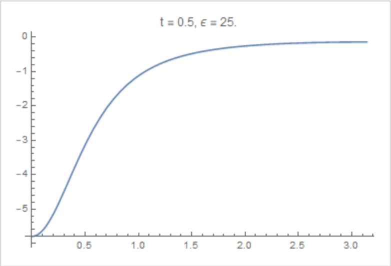
Separating delta-like terms yields:

$$\sigma_{1,b}^{(0)} = \frac{\varepsilon_1 - \varepsilon}{\varepsilon_1 + \varepsilon} \frac{q_0}{4\pi a_1 r_{01}} f\left(\frac{a_1}{r_{01}}, \mu\right) + \\ + \frac{\varepsilon_1 \varepsilon \left(\varepsilon_1 - \varepsilon\right)^2}{\left(\varepsilon_1 + \varepsilon\right)^3} \frac{q_0}{4\pi a_1 r_{01}} \sum_{n=0}^{\infty} \frac{\left(a_1/r_{01}\right)^n}{\left(n+1\right)\left[n\varepsilon_1 + \left(n+1\right)\varepsilon\right]} P_n\left(\mu\right).$$

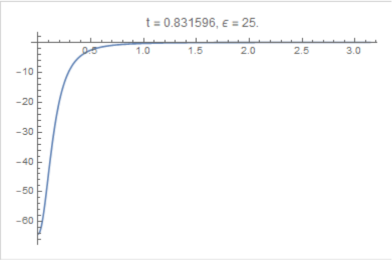
Here

$$f\left(t,\mu\right) = -\frac{1-t^2}{\left(1-2\mu t + t^2\right)^{3/2}} + \frac{2\varepsilon}{\varepsilon_1 + \varepsilon} \frac{1}{\sqrt{1-2\mu t + t^2}} + \\ + \frac{\varepsilon\left(\varepsilon_1 - \varepsilon\right)}{\left(\varepsilon_1 + \varepsilon\right)^2} \frac{1}{t} \ln \frac{t - \mu + \sqrt{1 - 2\mu t + t^2}}{1-\mu}.$$

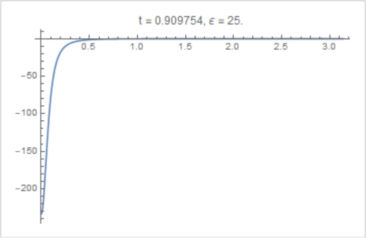
Induced charge density $f(\theta)$



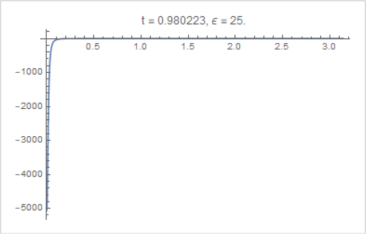
$r_{01} - a_1 = a_1$



$r_{01} - a_1 \approx a_1/5$



$r_{01} - a_1 \approx a_1/10$



$r_{01} - a_1 \approx a_1/50$

Electrostatic potential

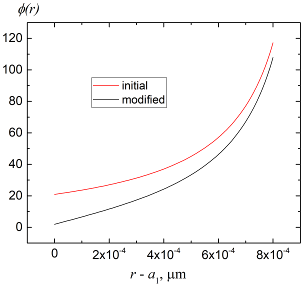
Similar manipulation of electrostatic potential

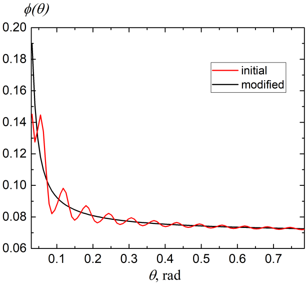
$$\phi_{out} = \frac{q_0}{\varepsilon r_0} + \frac{q_1}{\varepsilon r} - \sum_{n=1}^{\infty} \frac{n\left(\varepsilon_1 - \varepsilon\right)}{n\varepsilon_1 + (n+1)\,\varepsilon} \frac{q_0}{\varepsilon r_{01}^{n+1}} \frac{a_1^{2n+1}}{r^{n+1}} P_n\left(\mu\right)$$

gives the following expression:

$$\phi_{out} = \frac{q_0}{\varepsilon r_0} + \frac{q_1 + {q'}_0 - {q''}_0}{\varepsilon r} - \frac{{q'}_0}{\varepsilon {r'}_0} + \frac{{q''}_0}{\varepsilon {r'}_{01}} \ln \frac{{r'}_{01} + {r'}_0 - r\mu}{r\left(1-\mu\right)} + \\ + \frac{{q''}_0}{\varepsilon r} \sum_{n=1}^{\infty} \frac{\varepsilon_1}{(n+1)\left[n\varepsilon_1 + (n+1)\,\varepsilon\right]} \frac{a_1^{2n}}{r_{01}^{n} r^{n}} P_n\left(\mu\right).$$

Electrostatic potential





$$q_0 = 50e,\ q_1 = 100e,\ a_1 = 1\,\mu\text{m},\ \varepsilon_1 = 25,\ L = 10^{-3}\,\mu\text{m}.$$

- The two expressions are equivalent at large separation distances
- At short separations, the initial formula converges much slower than the modified one