New approach to the theory of void boundary for the rf discharge complex plasma

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Scientific-Coordination Session on “Non-Ideal Plasma Physics”
December 16-17, 2020

Instability threshold condition for a dust cloud

In the fluid approximation, the cloud dynamics is governed by the equations

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} + \nu\mathbf{v} = -\frac{\nabla p}{\mu} + \mathbf{e}f(\mathbf{r},t),$$

$$\frac{\partial \mu}{\partial t} + \nabla\cdot(\mu\mathbf{v}) = 0,$$

where $\mu\mathbf{e}f = \left[(3/8)(4\pi n_d/3)^{1/3} n_i \lambda e - Zen_d\right]\mathbf{E}$

is the sum of ion drag and ambipolar electric field force. From the ionization equation of state approach [D.I. Zhukhovitskii, Phys. Plasmas, 2019, v.26, p.063702], we have the particle charge equation

$$\theta\Phi^3 e^{\Phi} = \frac{n_e^*}{n_i^*}, \quad \theta = 2.8\tau^2 \frac{a}{\lambda_a}\left(\frac{T_e m_e}{T_i m_i}\right)^{1/2}$$

and the quasineutrality equation $1 - \frac{3}{4\pi}\frac{\Phi}{n_i^* \rho^3} = \frac{n_e^*}{n_i^*}$, where

$$n_e^* = (e^2\lambda_a^3 / aT_e)n_e, \quad n_i^* = (e^2\lambda_a^3 / aT_e)n_i, \quad \rho = r_d / \lambda_a, \ r_d = (3 / 4\pi n_d)^{1/3}.$$

For a stationary cloud $f = 0$, and we have $\frac{\pi}{2}\rho^2 n_i^* = \Phi\left(1 + \frac{3}{8\rho}\right), \ \Phi = Ze^2 / aT_e.$

We linearize the basic equations with $p = p_0 + p', \ \mu = \mu_0 + \mu'$:

$$\frac{\partial \mathbf{v}}{\partial t} + \nu\mathbf{v} = -\frac{\nabla p'}{\mu_0} + A\mathbf{e}\frac{\mu'}{\mu_0},$$

$$\frac{\partial \mu'}{\partial t} + \mu_0 \nabla \cdot \mathbf{v} = 0,$$

where $A = \mu_0 \partial f / \partial \mu$. We introduce $c_s^2 = dp / d\mu$ to derive from both equations

$$\nabla\left(\frac{\partial^2 \varphi}{\partial t^2} + \nu\frac{\partial \varphi}{\partial t} - c_s^2 \nabla^2 \varphi\right) = -A\mathbf{e}\nabla^2\varphi.$$

If we assume that $A = \mathrm{const}$, then for the case of a plane wave $\varphi \sim e^{i(\omega t \pm kx)}$,
the latter equation can be integrated, and we arrive at

$$\frac{\partial^2\varphi}{\partial t^2} + \nu\frac{\partial\varphi}{\partial t} - c_s^2\frac{\partial^2\varphi}{\partial x^2} + A\frac{\partial\varphi}{\partial x} = 0,$$

from which we derive the dispersion relation

$$c_s^2 k^2 - \omega^2 + i(\nu\omega \pm Ak) = 0 \quad \text{or} \quad \omega(k) \simeq c_s k + \frac{i}{2}\left(\nu \pm \frac{A}{c_s}\right).$$

The instability threshold condition, which we associate with the void boundary, is

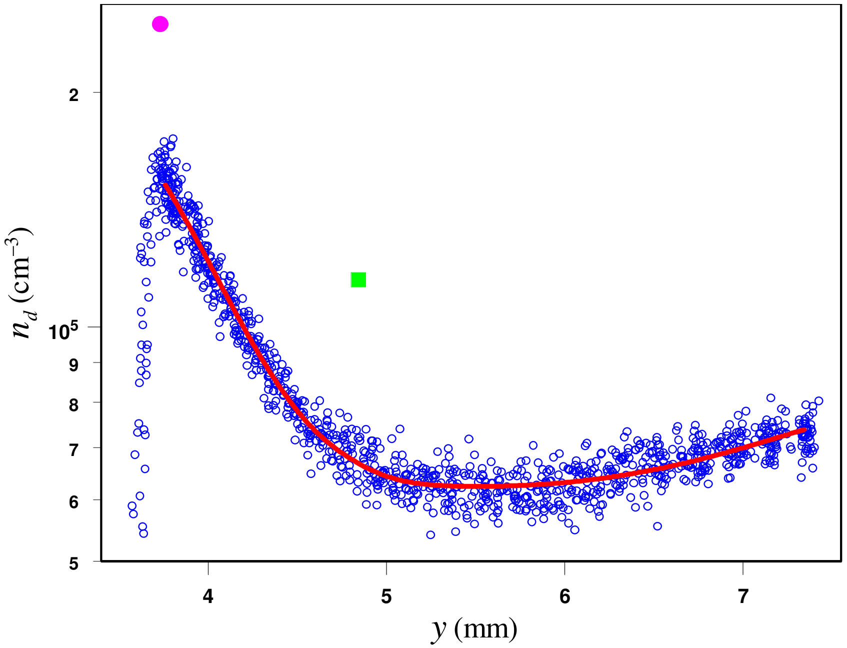
then $|A| > \nu c_s$ or $\dfrac{Zv_M^2}{3\nu c_s}\dfrac{3\gamma-2}{\gamma-1}\nabla \ln n_e > 1,$ where

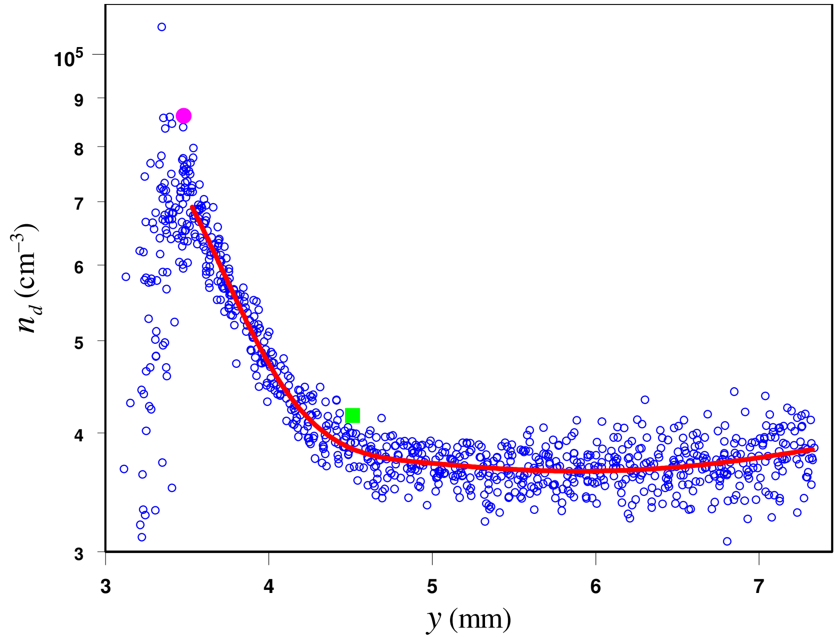
$$v_M^2 = T_e / M,\; Z = aT_e\Phi / e^2,\; \gamma = 1 - \theta\Phi^3 e^{\Phi}.$$

From the above criterion, it follows that (1), the void boundary position depends weakly on the gas pressure; (2), the distance from the discharge center decreases with the increase of the dust particle diameter; and (3), particles at the void boundary must strongly oscillate.

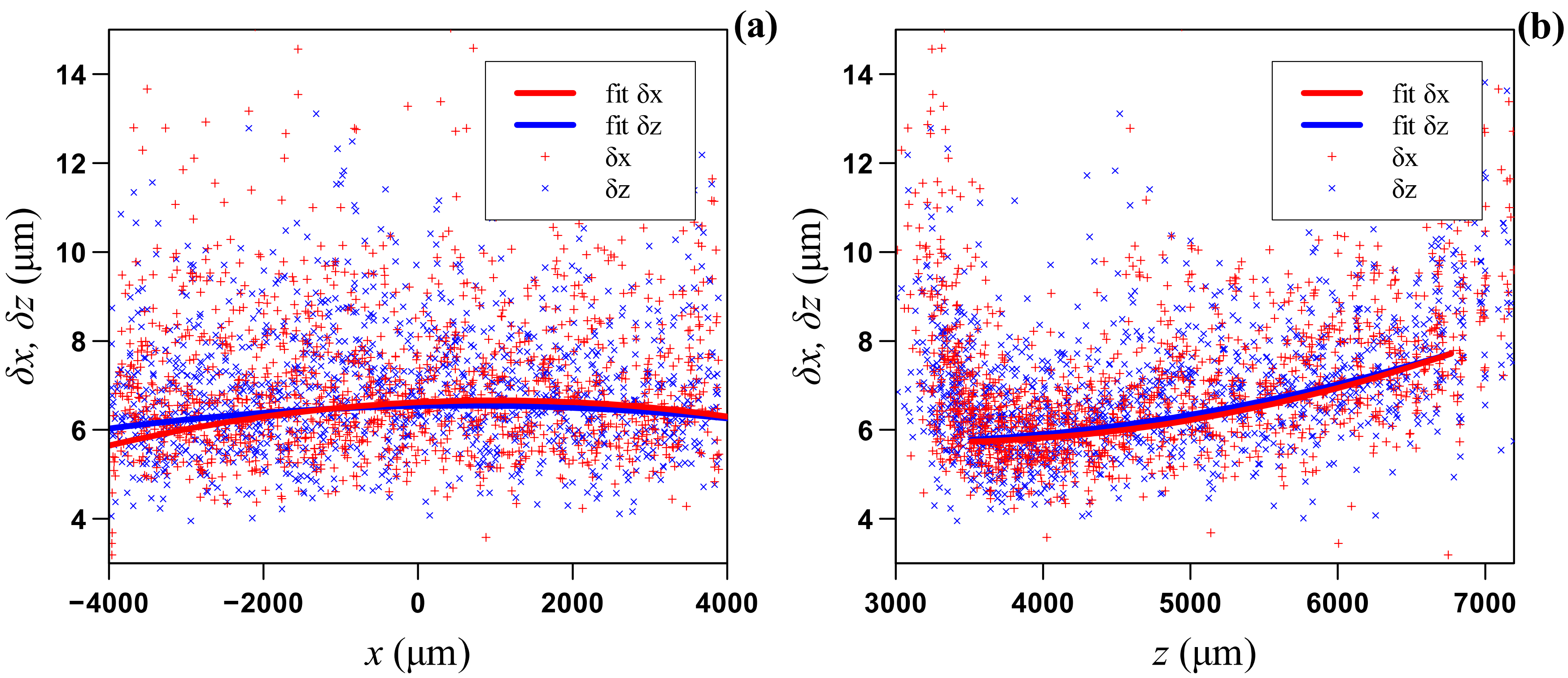
Dust number density distribution at fixed argon pressure $p_{\mathrm{Ar}} = 20.5$ Pa for

$2a = 3.42$ μm and $2a = 6.8$ μm.





Standard deviations of the dust particles from their equilibrium positions



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