ON THE CONDUCTIVITY OF MODERATELY NON-IDEAL COMPLETELY IONIZED PLASMA

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Interrelations between the problems of electrical conductivity of a completely ionized plasma and the ion drag force in a dusty plasma are discussed. It is shown that a physically motivated modifcation of the Coulomb logarithm proposed in the context of ion-particle scattering in dusty plasma allows us to improve the theoretical description of the conductivity in the moderately non-ideal regime. A simple theoretical expression obtained is in reasonable agreement with available results from experiments and numerical simulations.

Electrical conductivity of a completely ionized plasma is governed by electron-ion collisions. Elementary formula for the ideal plasma electrical conductivity is

$$\sigma \simeq \frac{e^2 n_e}{m_e \nu_{\rm eff}}, \qquad \nu_{\rm eff} = n_i v \sigma_s(v). \tag{1}$$

The classical momentum transfer cross section in the binary collision approximation is

$$\sigma_s(v) = 4\pi \int_0^{\rho_{\rm max}} \frac{\rho d\rho}{1+(\rho/\rho_0)^2} = 4\pi\rho_0^2\Lambda, \qquad \Lambda = \frac{1}{2}\ln\left(1+\frac{\rho_{\rm max}^2}{\rho_0^2}\right). \tag{2}$$

Here $\rho_0 = e^2/m_e v^2$ is the Coulomb radius and $\rho_{\rm max}$ is the maximum impact parameter. Traditionally, the velocity dependence under the Coulomb logarithm Λ is removed by assuming $m_e v^2 \simeq 3T_e$ and in the ideal (weakly coupled) regime, the maximum impact parameter is chosen as the electron Debye radius $\lambda_D = \sqrt{T_e/4\pi e^2 n_e}$. The Coulomb logarithm can then be expressed in terms of the coupling parameter $\Gamma = e^2/aT_e$, as $\Lambda = \ln(1+3/\Gamma^3)$, where $a = (4\pi n/3)^{-1/3}$ is the Wigner-Seitz radius. The condition of weak coupling $\Gamma \ll 1$ ensures that $\lambda_D \ll \rho_0$ and the Coulomb logarithm is large. In this case we get approximately $\Lambda \simeq \ln(\sqrt{3}/\Gamma^{3/2})$.

The specifics of dusty plasmas is that the grain charge is normally rather high and the condition $\Lambda_D \ll \rho_0$ is usually not satisfied. In this case, it is not sufficient to consider the ions with impact parameters below Λ_D, because there exist considerable fraction of ions which can approach close to the particle even if the impact parameter is larger than Λ_D. In terms of electron-ion collisions this situation corresponds to the regime $\Gamma \gtrsim 1$. A modification to the conventional Coulomb scattering theory in this regime has been put forward in Ref. [1]. Here it was proposed to take into account all the ions that approach the grain closer than the Debye radius. The modification mainly affects the Coulomb logarithm, which becomes

$$\Lambda = \ln\left(1 + \frac{\lambda_D}{\rho_0}\right). \tag{3}$$

This modified Coulomb logarithm reduces to the conventional one in the weak coupling limit $\lambda_D \ll \rho_0$, but provides much better estimate of the momentum transfer cross section at $\lambda_D \sim \rho_0$.

The suggested modification is expected to provide better accuracy at $e^2/T_e\lambda_D \sim \Gamma^{3/2} \sim \mathcal{O}(1)$. Since in this regime the argument of the logarithm is not large, velocity dependence under the logarithm should not be omitted. Performing usual steps of the derivation we arrive for the effective electron collision frequency with the modified Coulomb logarithm

$$\nu_{\text{eff}} = \frac{4\sqrt{2\pi}n_e e^4 \Lambda_{\text{mod}}}{3\sqrt{m_e}T_e^{3/2}}, \qquad \Lambda_{\text{mod}} = 3\left[\int_0^\infty \frac{x^7 e^{-x^2}dx}{\ln\left(1 + \frac{2x^2}{\sqrt{3\Gamma^3}}\right)}\right]^{-1}. \tag{4}$$

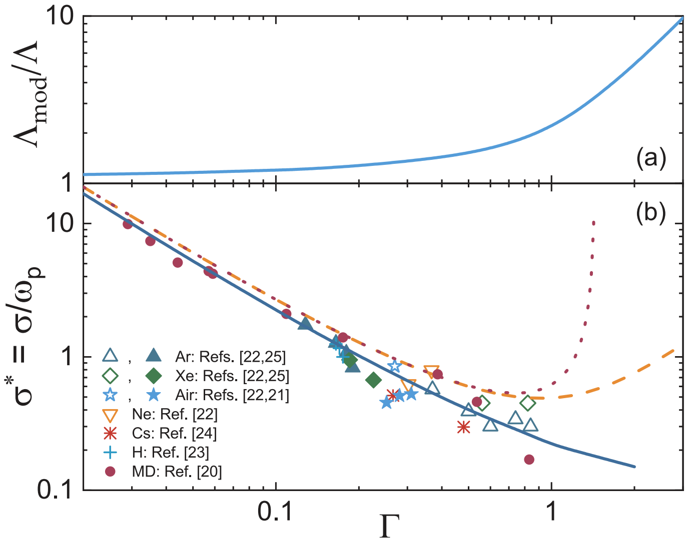

FIG. 1. Ratio of the modified and conventional Coulomb logarithms (a). Reduced conductivity of a moderately non-ideal ($\Gamma \sim 0.02 - 2$) fully ionized plasma as a function of the coupling parameter Γ (b). Symbols correspond to experimental results and MD simulations (see the legend in Ref. [2]). Dotted line corresponds to the weakly coupled limit of the conventional Coulomb logarithm, dashed line to the unsimplified conventional Coulomb logarithm, and the blue solid line to the modified Coulomb logarithm of Eq. (4).

The modified and conventional Coulomb logarithms. It is observed that, as expected, they nearly coincide at weak coupling, $\Gamma \ll 1$, whereas the modified version becomes considerably larger at $\Gamma \gtrsim 1$.

Comparison of the obtained results on conductivity with available experimental and MD simulation results shown in Fig. 1 demonstrates that in the ideal plasma limit ($\Gamma \ll 1$) the difference between the conventional and modified approaches vanishes. Here the Coulomb logarithm of the simple weakly coupled form $\Lambda = \frac{1}{2}\ln(3/\Gamma^3)$ does a very good job. On approaching the moderately non-ideal regime with $\Gamma \sim 1$, this simple form predicts the conductivity divergence and should not be used. The use of the non-simplified Coulomb logarithm $\Lambda = \frac{1}{2}\ln(1 + 3/\Gamma^3)$ allows one to avoid divergence, but this form still somewhat overestimates most of the experimental and MD data. The modified Coulomb logarithm provides better agreement with experiments and simulations. The proposed simple model provides a useful tool for semi-quantitative conductivity estimates in the moderately non-ideal plasma regime.

More detail information one can find in Ref. [2].

References

1. Khrapak S.A., Ivlev A.V., Morfill G.E., Thomas H.M., Physical Review E **66**, 046414 (2002).

2. Khrapak S.A., Khrapak A.G., Results in Physics **17**, 103163 (2020)