A field-theoretic approach to the study of parametric instability of a quantum metamaterial

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Excitations in classical non-linear media pose non-trivial properties. In particular, propagating waves may be subject to instability. It is a challenging problem to study such effects in a quantum case. One possible system where such a scenario may take place is a set of Josephson junctions organized into a superconducting travelling wave parametric amplifier (TWPA). This device can be considered as a non-linear metamaterial. If a coherent drive is applied to the TWPA, quantum effects result in radiation generation in a wide frequency band. It is impossible to describe this process within the framework of a classical approach, but we were able to derive a quantum Hamiltonian and use it in a field-theoretic approach based on the Schwinger-Keldysh technique in Nambu representation similar to [1]. We managed to derive quantum kinetic equations for normal and anomalous photon occupation numbers of the modes of the generated radiation. We show that the spectrum of generated radiation strongly depends on the spectrum of TWPA and driving frequency. Moreover, under specific conditions non-trivial regime of generation arize. This regime is characterized by an exponential increase in generation power and resembles parametric instability in classical non-linear media. The transition from "ordinary" generation to this regime is similar to spectral phase transition [2].

[1] Buchhold M and Diehl S 2015 *Eur. Phys. J. D* **69**

[2] Roy A, Jahani S, Langrock C, Fejer M and Marandi A 2021 *Nat. Commun.* **12**