A simplified mathematical model for deflagration-to-detonation transition

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In this work, we investigate a model proposed in [1] that contains simplified equations for both flames and detonations in certain limits of its parameters. The equations are constructed such that in the limit of detonations, the model reduces to an asymptotic model of weakly nonlinear detonations [2]. In the opposite limit of flames, the model reduces to a reaction-diffusion system of combustion theory. We show here numerically that the combined model predicts both slow flames and detonations as well as a transition between them. The model consists of the following system of partial differential equations: (1) $u_t + u_x + \varepsilon u u_x = -\frac{\varepsilon}{2} T_x + \frac{4\delta}{3\varepsilon} \mathrm{Pr} u_{\mathrm{xx}}$; (2) $T_t - u_t = q\omega + \delta T_{\mathrm{xx}}$; and (3) $\lambda_t = \omega + \frac{\delta}{\mathrm{Le}} \lambda_{\mathrm{xx}}$. The heat release rate is given by Arrhenius law $\omega = k\left(1-\lambda\right)\exp\left(-\vartheta/T\right) H\left(T - T_{\mathrm{ign}}\right)$ with activation energy ϑ, ignition temperature T_{ign}, and the Heaviside function H. We also assume ε, $\delta \ll 1$ and Pr, Le, q, $\vartheta \sim 1$. An ignition problem by a hot spot at the closed end of a tube was analyzed. For weak sources, we have found a traveling flame front, while for strong sources–a detonation wave. Furthermore, the existence of spontaneous transitions from a subsonic traveling wave to a supersonic one accompanied by a sharp release of energy is also found, confirming that the model at hand predicts the DDT process.

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[1] Faria L M, Lau-Chapdelaine S, Kasimov A R and Rosales R R 2017 A toy model for detonations and flames *26th International Colloquium on the Dynamics of Explosions and Reactive Systems (ICDERS)*

[2] Faria L M, Kasimov A R and Rosales R R 2015 *Journal of Fluid Mechanics* **784** 163–198 (*Preprint* http://arxiv.org/abs/1407.8466)