Simulation of Richtmyer-Meshkov instability in solids via smoothed particle hydrodynamics

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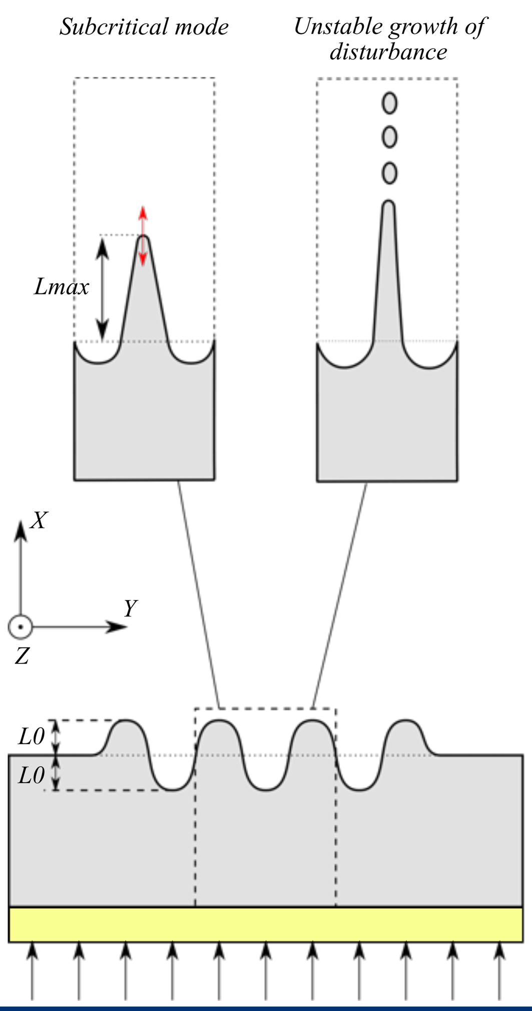
Introduction

Shock loading of solid samples with grooved surface may result in development of cumulative jets usually associated with Richtmyer-Meshkov (RM) instablility. This kind of instability along with the Rayleigh-Taylor one plays a crucial role in inertial confinement fusion [1], as well as on experimental facilities for high energy density physics. In the last decade study of such phenomena has attained a lot of attention resulting in careful experimental, numerical, and theoretical studies.

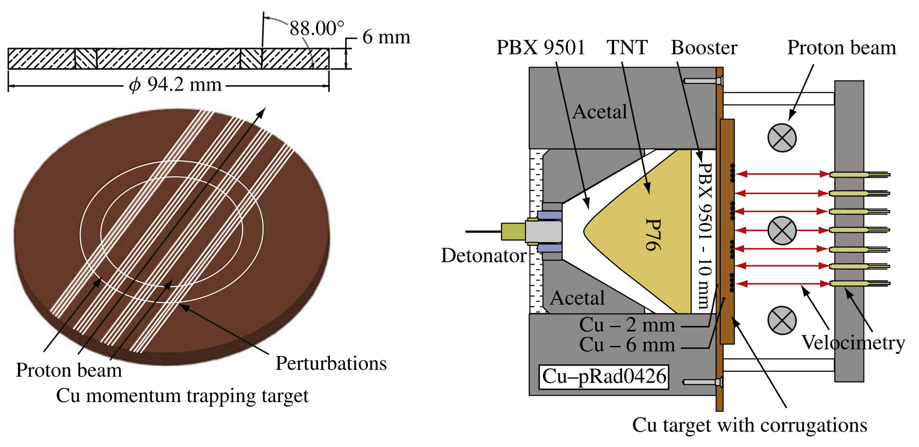
Starting with experimental work of Barnes et al. [2], the analysis of the RM-instability development has become one of the methods for determining the strength of materials at high strain rates. One may identify 3 regimes of shock wave (SW) loading:
1) light SW, leading to elastic strain and surface oscillations;
2) average SW, leading to plastic flow and arrested growth of jets;
3) strong SW, leading to unstable material flow from melt surface.

By performing simulations we aimed to develop a proper material model for regimes 1 and 2. Our simulation setup is based on the experimental studies [3,4]. The tantalum and copper model parameters are fitted to achieve best agreement with experiment thus providing the proper models for high strain rate loading.

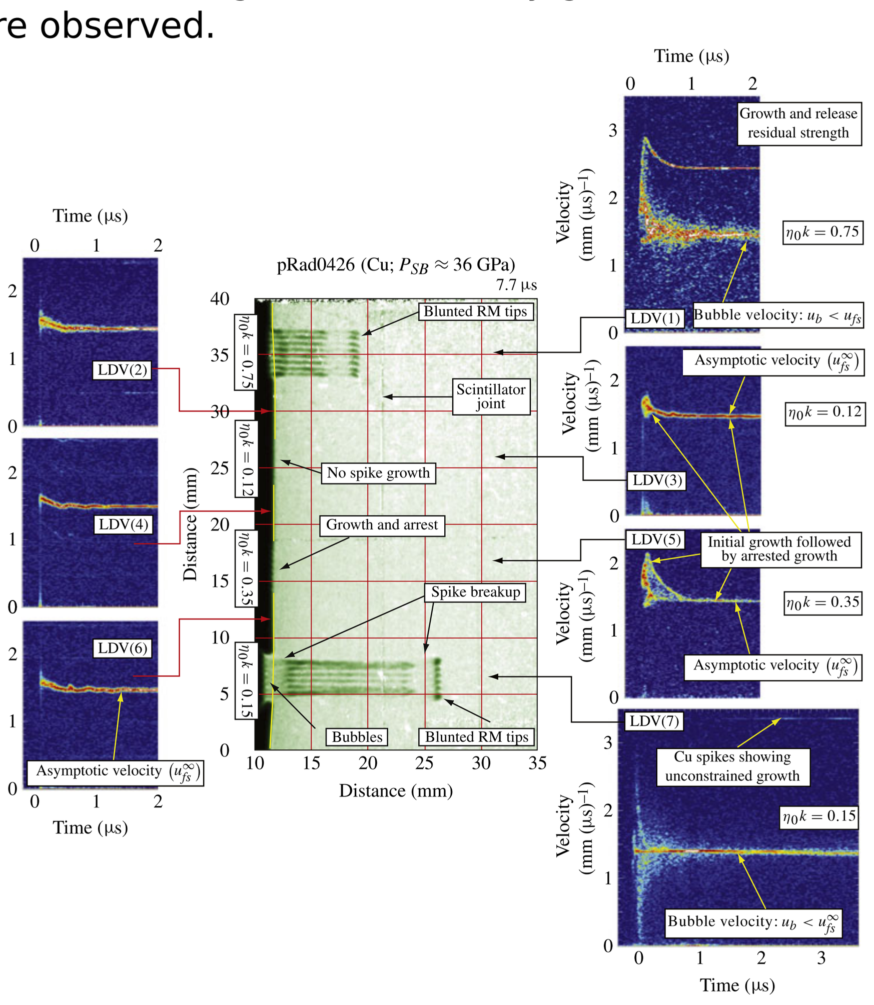


Setup and results of the experiments [3,4]

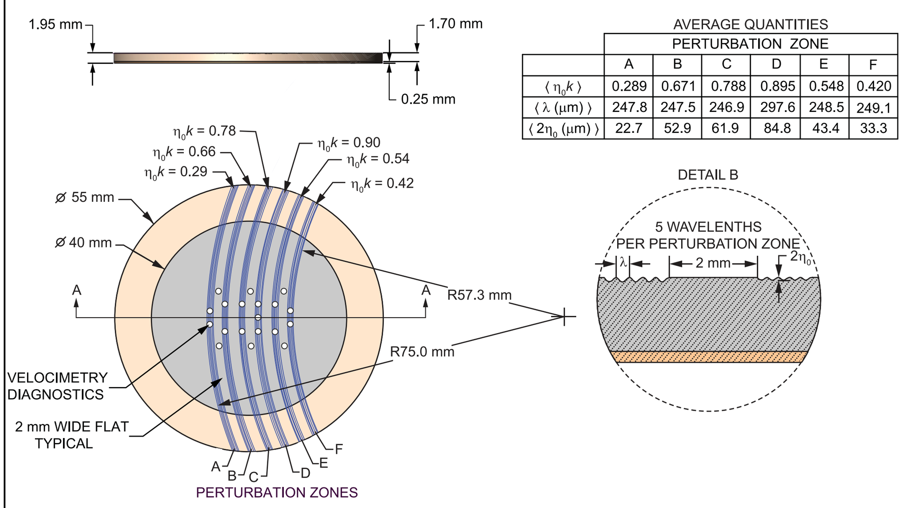
Copper [3]

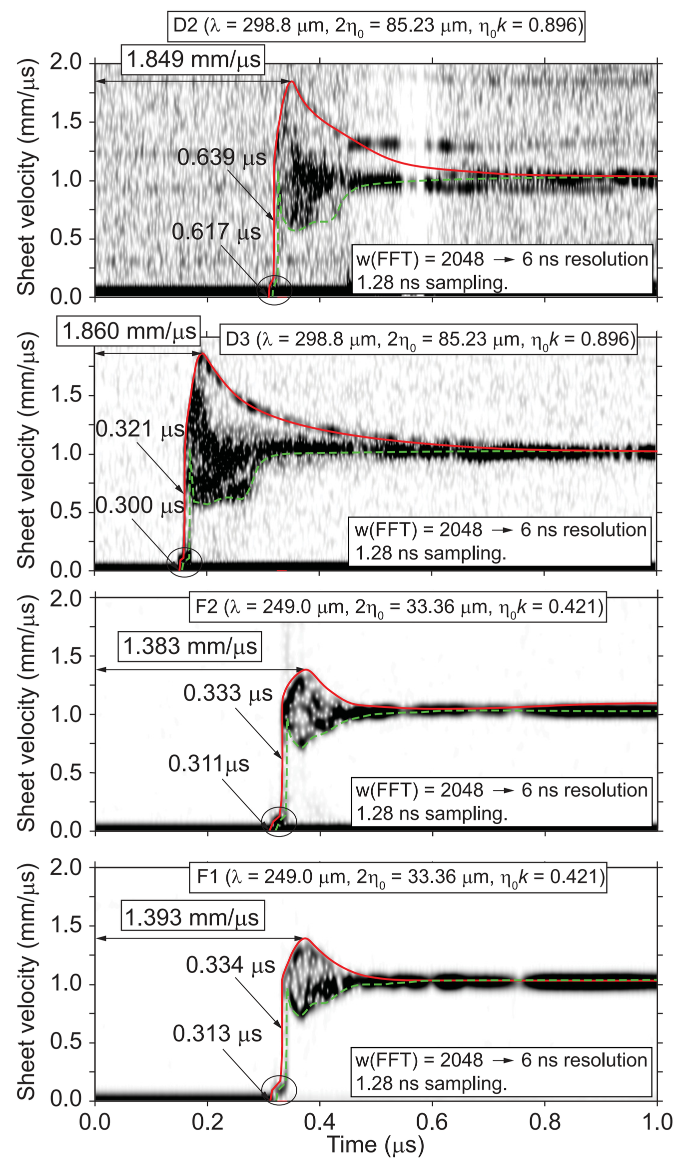


Targets with regular corrugations are subjected to a shock loading. Diagnostics includes proton radiography and photon (laser) doppler velocimetry. Spikes are ejected from deep corrugations, while for small corrugations instability growth and arrest are observed.



Tantalum [4]

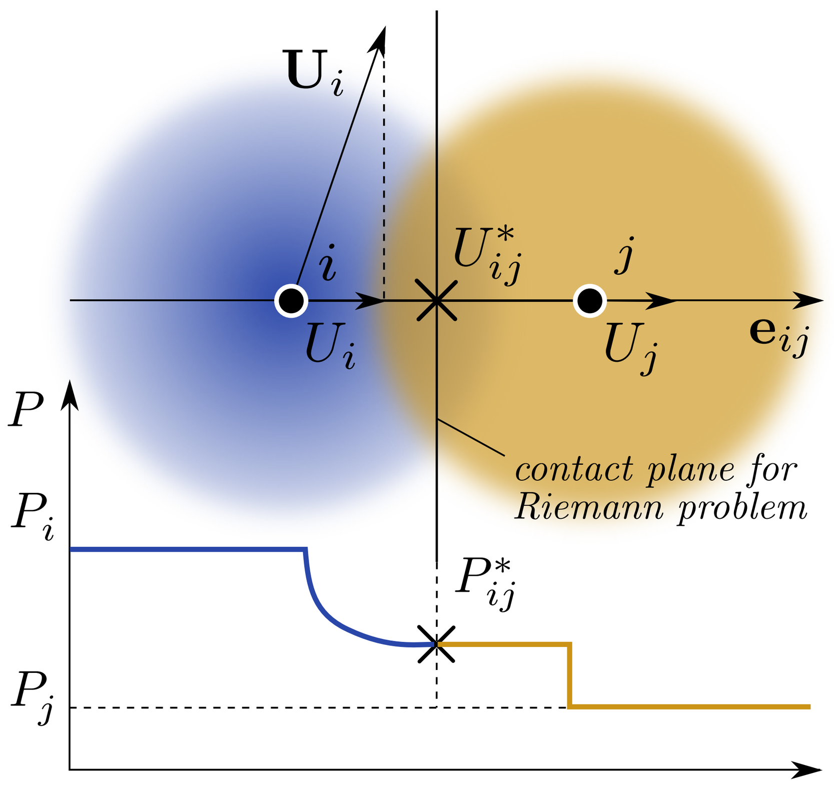




Theoretical and numerical models

Oscillations [5,6] (regime 1)	Arrested growth [6] (regime 2)	Unstable growth [7] (regime 3)
$T = \frac{2\pi}{\omega} = \alpha\lambda\sqrt{\frac{\rho}{2G}}$	$\eta_{spike} \simeq \alpha\frac{\rho v^2}{kY}$	$\eta_0 > \eta_{cr} \simeq \alpha\frac{Y}{\rho a}$

Contact SPH with strength [8,9]



smoothed particles approximation

$$f(\mathbf{r}) = \sum_{j=1}^{N} f_j \Delta V_j W(|\mathbf{r} - \mathbf{r}_j|, h)$$

smoothed particles hydrodynamics

$$\frac{d\rho_i}{dt} = -2\rho_i \sum_{j=1}^{N} \frac{m_j}{\rho_j}(v_i^R - v_{ij}^{*R})W'_{ij},$$

$$\frac{d\mathbf{v}_i}{dt} = -\frac{2}{\rho_i}\sum_{j=1}^{N}\frac{m_j}{\rho_j}\,\sigma_{ij}^* W'_{ij},$$

$$\frac{de_i}{dt} = -\frac{2}{\rho_i}\sum_{j=1}^{N}\frac{m_j}{\rho_j}(\sigma_{ij}^*\cdot\mathbf{v}_{ij}^*)W'_{ij}$$

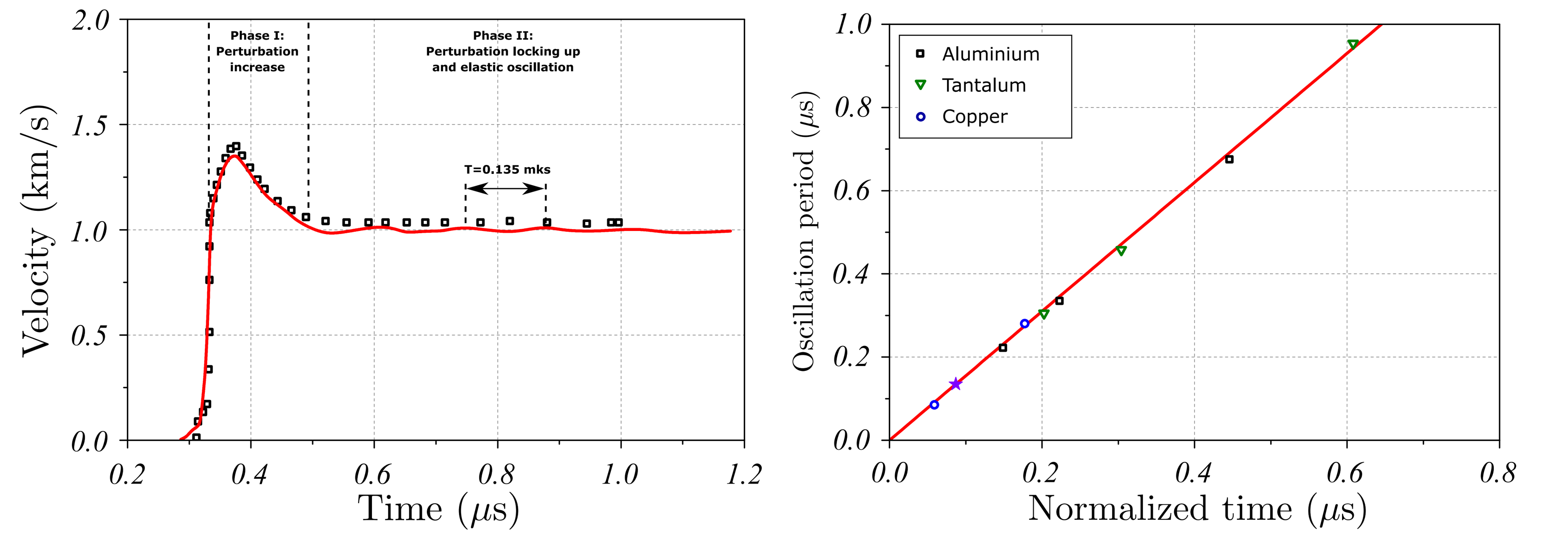
Stress deviator evaluation

$$\frac{dS_i^{\alpha\beta}}{dt} = -2G\sum_{j=1}^{N}\frac{m_j}{\rho_j}\left[(v_i^\alpha - v_{ji}^{*\alpha})e_{ij}^\beta + (v_i^\beta - v_{ji}^{*\beta})e_{ij}^\alpha + \frac{2}{3}\delta^{\alpha\beta}(v_i^R - v_{ij}^{*R})\right]W'_{ij}$$

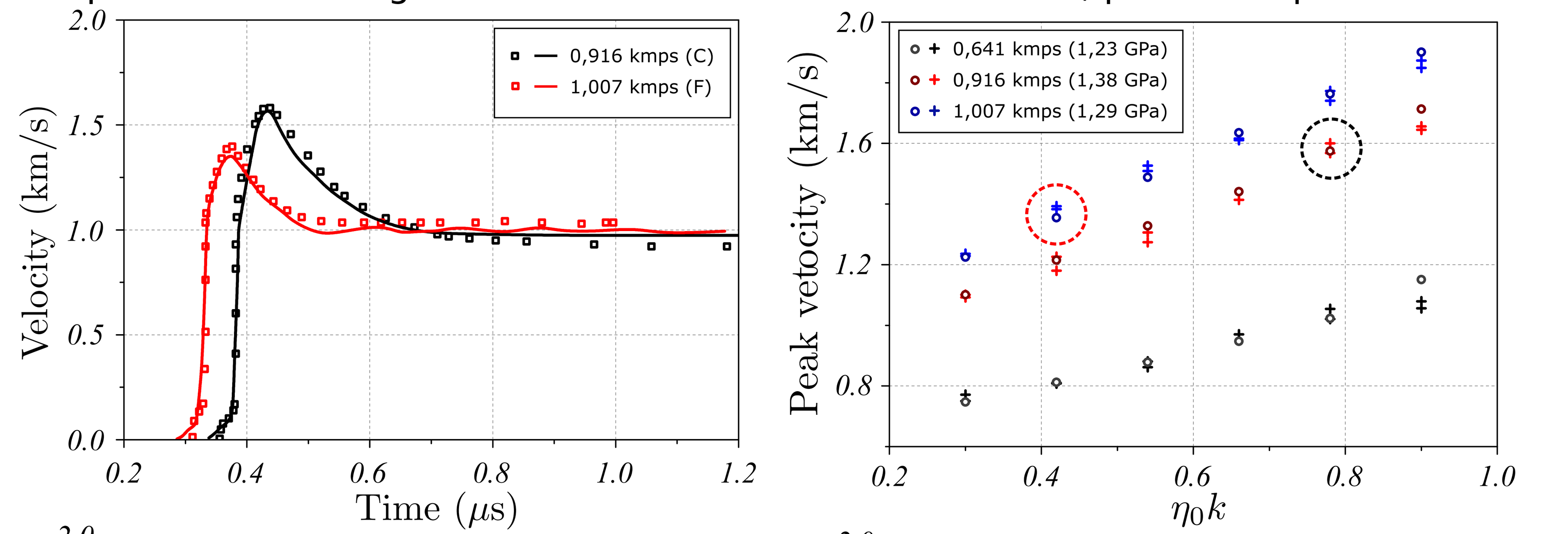
EoS $P - P_r = \gamma\rho(e - e_r) \qquad u_s = c + su_p$

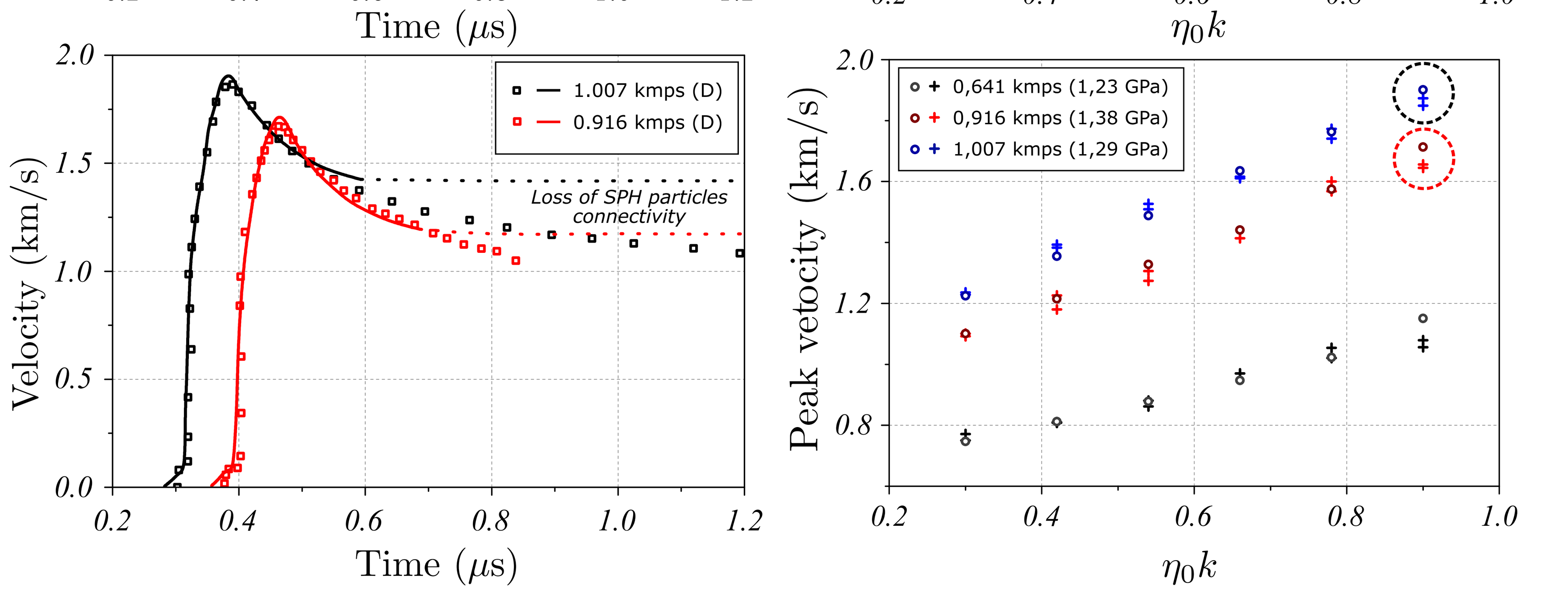
Simulation results

Regimes 1~2: arrest and oscillations



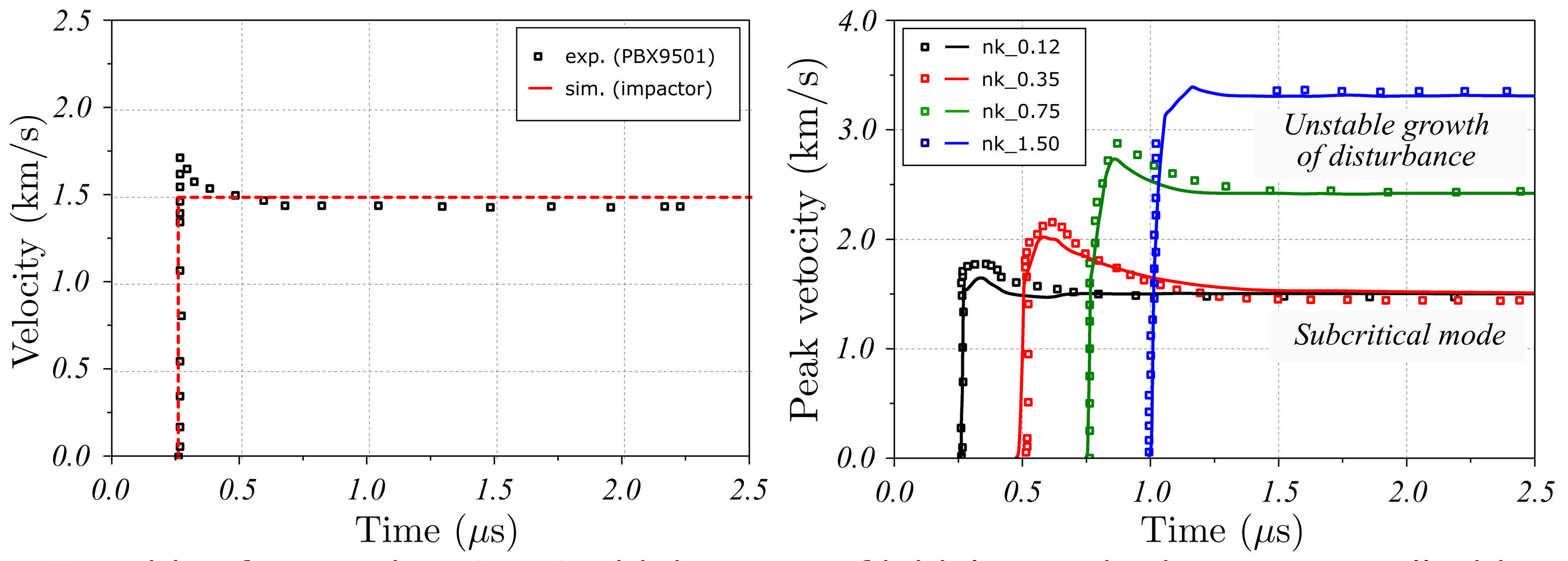
Oscillations are observed after spike growth and arrest. Period of oscillations can be expressed as for regime 1 with $\alpha = 1.55$. Solid lines SPH, points - experiment.





Peak velocities are in good agreement with the experiment [4].

Regimes 2~3: arrest or unstable growth



Transition from regime 2 to 3 with increase of initial perturbation agrees well with the experiment [3]. Solid lines - SPH, points - the experiment [3].

Conclusion

In this study we have simulated the behavior of copper and tantalum at high strain rates based on the experiments [3,4] with our CSPH method. The strength parameters (yield strength, shear modulus) are adjusted to fit the experimental spike velocity profile and can be found in the table. Such approach seems promising for strength parameters evaluation at high strain rates.

Strength and EoS parameters

Mechanical properties	Al	Cu	Ta
Initial density ρ_0, kg/m^3	2610	8960	16656
Shear modulus G, GPa	26.22	43.0	69.0
Yield strength Y, GPa	0.18	0.53	1.23-1.38
Equation of state parameters			
Parameter Γ	2.17	2.0	2.0
Parameter c, km/s	5.35	3.958	3.43
Parameter s	1.35	1.497	1.19

The suppression regime is investigated by SPH simulations of Richtmyer-Meshkov instability in solid tantalum [4]. It is shown that for the yield strength of tantalum proposed in the original work [4], the simulation results reproduce well the maximum spike velocity obtained in experiments. Good agreement indicates the possibility of using the contact SPH method to determine the strength materials at high strain rates.

The regime of unstable growth of perturbations is studied for copper [3]. Good qualitative and quantitative agreement between the simulation results and experimental results is obtained. The simulation reproduces both the suppression effect in the unstable growth regime and the asymptotic velocity. The increase in the jet velocity with an increase in the amplitude of the initial disturbances is reproduced with good accuracy.

References

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